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General Fault Admittance Method Line-To-Line-To-Ground Faults In Reference And Odd Phases

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Abstract
Line-to-line-to-ground faults are usually analysed using connection of symmetrical component networks at the fault point. As a first step, a reference phase is chosen which results in the simplest connection of the symmetrical component sequence networks for the fault. The simplest connection of symmetrical component sequence networks is a parallel one of the positive, negative and zero sequence networks when phase *a* of an *abc* phase sequence is the reference phase and the fault is taken to be between the *b* and *c* phases and ground. Putting the fault on an odd phase, say between the *a* and *c* phases and ground results in a parallel connection of the positive, negative and zero sequence networks that involve phase shifts, and the solution is more demanding. In practice, the results for the line-to-line-to-ground fault for the reference phase a may be translated to a fault on odd phases by appropriate substitution of phases. In this approach, the solution proceeds by assuming that the fault is in phases *b* and *c* and ground and that the symmetrical sequence networks are connected in parallel. The solution of the fault on *b* and *c* phases and ground is then translated to apply to the fault on odd phases, say either between phases *c* and *a* and ground or between phases *a* and *b* and ground. Alternatively, the parallel connection of the sequence networks at the fault point for the odd phases fault is solved for the symmetrical component currents and voltages. These are then used to determine the symmetrical component voltages at the other busbars and hence the symmetrical component currents in the rest of the system. The connection of the sequence networks must be known for the common fault types. In contrast, a solution by the general method of fault admittance matrix does not require prior knowledge of how the sequence networks are connected. A fault may be on any two phases, the double line-to-ground fault on the reference phase or on odd phases, and a solution is obtained for the particular fault. It is therefore more versatile than the classical methods since it does not depend on prior knowledge of how the sequence networks are connected. The paper presents solutions for line-to-line-to-ground faults on the reference and on odd phases of a simple power system containing a delta-earthed-star connected transformer. The results, which include the effects of the delta-earthed-star connected transformer, show that the general fault admittance method solves line-to-line-to-ground faults on odd phases.
***Keywords* -** Line to line to ground fault on odd phases, Unbalanced faults analysis, Fault admittance matrix, Delta-earthed-star transformer.

I. INTRODUCTION

The paper shows that the general fault admittance method of fault analysis may be used to solve line to line to ground faults on odd phases. The method does not require one to have a good understanding of how the sequence networks are connected in the classical approach, so that one may interpret the results obtained for the reference phase fault to the odd phases.

The general fault admittance method differs from the classical approaches based on symmetrical components in that it does not require prior knowledge of how the sequence components of currents and voltages are related [1-3]. In the classical approach, knowledge of how the sequence components are related is required because the sequence networks have to be connected in a prescribed way for a particular fault. Then the sequence currents and voltages at the fault are determined, after which symmetrical component currents and voltages in the rest of the network are calculated. Phase currents and voltages are found by transforming the respective symmetrical component values [3-11].

Another consideration is that in the classical analysis common faults have reference faults that are solved and then the results applied to odd phase faults. For example line-to-line-to-ground faults are always solved with reference to the *a* phase, in an *abc* phase system, or its equivalent. The fault is put between phases *b* and *c* and ground. It is known that for this type of fault $V_1 = V_2 = V_0$ and that $I_1 + I_2 + I_0 = 0$, where the variables *V* and *I* refer to voltage, current respectively and the subscripts *1*, *2* and *0* refer to the positive, negative and zero sequence components respectively.

The solution is obtained in relation to the reference phase *a*. However, when the line-to-line-to-ground fault is on the odd phase, either on the *b* or *c* phase, i.e. the line to line fault either involves phases *c* and *a* or *a* and *b* phases, the results for the reference phase line to line to ground fault are transformed to the odd phase. That is the results are interpreted in respect of the odd phases fault reference, which may be *b* or *c*, taking into account the symmetrical component constraints. The reference phase is replaced by the odd phase reference and the results converted accordingly. Table 1 shows the voltage and current symmetrical component constraints for line-to-line-to-ground faults on the reference phase *a* and on odd phases *b* and *c*. In Table 1, the complex operator $\alpha=1\angle 120^\circ$.

Table 1: Symmetrical component constraints for Line-to-Line-to-Ground Faults.

Reference Phase	**Odd Phases**	
a	*b*	*c*
$V_{a1}=V_{a2}=V_{a0}$ $V_1=V_2=V_0$	$V_{b1}=V_{b2}=V_{bo}$ $\alpha^2 V_{a1}=\alpha V_{a2}=V_{a0}$ $\alpha^2 V_1=\alpha V_2=V_0$	$V_{c1}=V_{c2}=V_{co}$ $\alpha V_{a1}=\alpha^2 V_{a2}=V_{a0}$ $\alpha V_1=\alpha^2 V_2=V_0$
$I_{a1}+I_{a2}+I_{a0}=0$ $I_1+I_2+I_0=0$	$I_{b1}+I_{b2}+I_{bo}=0$ $\alpha^2 I_{a1}+\alpha I_{a2}+I_{a0}=0$ $\alpha^2 I_1+\alpha I_2+I_0=0$	$I_{c1}+I_{c2}+I_{co}=0$ $\alpha I_{a1}+\alpha^2 I_{a2}+I_{a0}=0$ $\alpha I_1+\alpha^2 I_2+I_0=0$

The fault admittance method is general in the sense that any fault impedances may be represented, provided the special case of a zero impedance fault is catered for. Therefore, a line-to-line-to-ground fault with the reference on an odd phase, say on the *b* or *c* phases, poses no difficulties and is easily accommodated.

This paper presents the results of line-to-line-to-ground faults on the references of phase *a* and odd phases *b* and *c* of a simple power system obtained using the general fault admittance method.

II. BACKGROUND

Sakala and Daka [1-3] and Elgerd [4] discussed the solution procedure of the general fault admittance method. However, it is presented here in brief, showing the salient features, key equations and the solution procedure.

A line-to-line-to-ground fault presents low value impedances, with zero value for a direct short circuit or metallic fault, between the faulted two phases and ground at the point of fault in the network. In general, a fault may be represented as shown in Figure 1.

In Figure 1, a fault at a busbar is represented by fault admittances in each phase, i.e. the inverse of

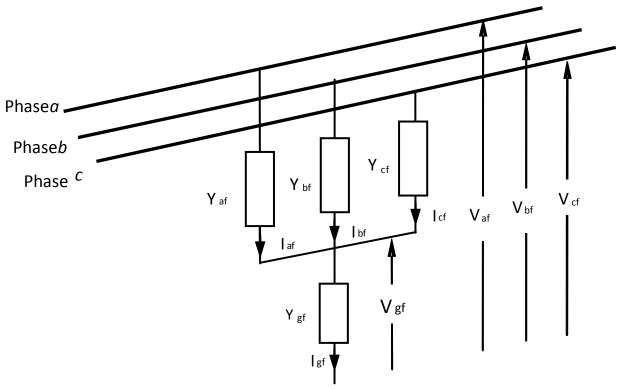

Figure 1: General Fault Representation

the fault impedance in the phase, and the admittance in the ground path. Note that the fault admittance for a short-circuited phase is represented by an infinite value, while that for an open-circuited phase is a zero value. Thus, for a line-to-line-to-ground fault the admittances Y_{bf} and Y_{cf} and Y_{gf} are infinite while on reference phase *a*, the admittance for Y_{af} is infinite.

The general fault admittance matrix is given by:

$$Y_f = \left(\frac{1}{Y_{af}+Y_{bf}+Y_{cf}+Y_{gf}}\right) \times \begin{bmatrix} Y_{af}\left(Y_{bf}+Y_{cf}+Y_{gf}\right) & -Y_{af}Y_{bf} & -Y_{af}Y_{cf} \\ -Y_{af}Y_{bf} & Y_{bf}\left(Y_{af}+Y_{cf}+Y_{gf}\right) & -Y_{bf}Y_{cf} \\ -Y_{af}Y_{cf} & -Y_{bf}Y_{cf} & Y_{cf}\left(Y_{af}+Y_{bf}+Y_{gf}\right) \end{bmatrix} \quad (1)$$

Equation (1) is transformed using the symmetrical component transformation matrix *T*, and its inverse T^{-1}, where

$$T = \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} \text{ and } T^{-1} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix},$$

in which $\alpha = 1\angle 120^\circ$ is a complex operator.

The symmetrical component fault admittance matrix is given by the product:

$$Y_{fs} = T^{-1} Y_f T$$

The general expression [1-3] for Y_{fs} is given by:

$$Y_{fs} = \frac{1}{Y_{af}+Y_{bf}+Y_{cf}+Y_{gf}} \begin{bmatrix} Y_{fs11} & Y_{fs12} & Y_{fs13} \\ Y_{fs21} & Y_{fs22} & Y_{fs23} \\ Y_{fs31} & Y_{fs32} & Y_{fs33} \end{bmatrix} \quad (2)$$

where

$$Y_{fs11} = Y_{fs22} = \frac{1}{3} Y_{gf}\left(Y_{af}+Y_{bf}+Y_{cf}\right) + Y_{af}Y_{bf} + Y_{af}Y_{cf} + Y_{bf}Y_{cf}$$

$$Y_{fs33} = \tfrac{1}{3} Y_{gf}\left(Y_{af}+Y_{bf}+Y_{cf}\right)$$

$$Y_{fs12} = \tfrac{1}{3} Y_{gf}\left(Y_{af}+\alpha^2 Y_{bf}+\alpha Y_{cf}\right) - \left(Y_{bf}Y_{cf}+\alpha Y_{af}Y_{bf}+\alpha^2 Y_{af}Y_{cf}\right)$$

$$Y_{fs21} = \tfrac{1}{3} Y_{gf}\left(Y_{af}+\alpha Y_{bf}+\alpha^2 Y_{cf}\right) - \left(Y_{bf}Y_{cf}+\alpha^2 Y_{af}Y_{bf}+\alpha Y_{af}Y_{cf}\right)$$

$$Y_{fs13} = Y_{fs32} = \tfrac{1}{3} Y_{gf}\left(Y_{af}+\alpha Y_{bf}+\alpha^2 Y_{cf}\right) \text{ and}$$

$$Y_{fs31} = Y_{fs23} = \tfrac{1}{3} Y_{gf}\left(Y_{af}+\alpha^2 Y_{bf}+\alpha Y_{cf}\right)$$

The above expressions simplify considerably depending on the type of fault. For example, for a line to line fault:

$Y_{af} = Y_{gf} = 0,\quad Y_{bf} = Y_{cf} = 2Y,$ i.e. $Z_{af} = Z_{gf} = \infty$

$$Y_{fs} = \frac{1}{2Y+2Y}\begin{bmatrix} 2Y\times 2Y & -(2Y\times 2Y) & 0 \\ -(2Y\times 2Y) & 2Y\times 2Y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= Y\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3)$$

For a line-to-line-to-ground fault:

$Y_{af} = 0,\ Y_{bf} = Y_{cf} = 2Y,\quad Y_{gf} = \infty$ i.e. $Z_{af} = \infty$

$$Y_{fs} = \frac{1}{2Y_f + Y_{gf}}\begin{bmatrix} \frac{2}{3}Y_{gf}Y + Y^2 & -\frac{1}{3}Y_{gf}Y - Y^2 & -\frac{1}{3}Y_{gf}Y \\ -\frac{1}{3}Y_{gf}Y - Y^2 & \frac{2}{3}Y_{gf}Y + Y^2 & -\frac{1}{3}Y_{gf}Y \\ -\frac{1}{3}Y_{gf}Y & -\frac{1}{3}Y_{gf}Y & \frac{2}{3}Y_{gf}Y \end{bmatrix} \quad (4a)$$

When $Y_{gf} = Y$ i.e. fault impedance in ground path equal fault impedance in faulted phases equation 4a becomes:

$$Y_{fs} = \frac{Y}{9}\begin{bmatrix} 5 & -4 & -1 \\ -4 & 5 & -1 \\ -1 & -1 & 2 \end{bmatrix} \quad (4b)$$

2.1 Currents in the Fault

At the faulted busbar, say busbar j, the symmetrical component currents in the fault are given by:

$$I_{fsj} = Y_{fs}\left(U + Z_{sjj}Y_{fs}\right)^{-1}V_{sj}^{\ 0} \quad (5)$$

where U is the unit matrix:

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and Z_{sjj} is the jj^{th} component of the symmetrical component bus impedance matrix:

$$Z_{sjj} = \begin{bmatrix} Z_{sjj+} & 0 & 0 \\ 0 & Z_{sjj-} & 0 \\ 0 & 0 & Z_{sjj0} \end{bmatrix}$$

The element Z_{sjj+} is the Thevenin's positive sequence impedance at the faulted busbar, Z_{sjj-} is the Thevenin's negative sequence impedance at the faulted busbar, and Z_{sjj0} is the Thevenin's zero sequence impedance at the faulted busbar.

Note that as the network is balanced the mutual terms are all zero.

In equation (5) V_{sj}^{0} is the prefault symmetrical component voltage at busbar j the faulted busbar:

$$V_{sj}^{\ 0} = \begin{bmatrix} V_{sj+} \\ V_{sj-} \\ V_{sj0} \end{bmatrix} = \begin{bmatrix} V_+ \\ 0 \\ 0 \end{bmatrix}$$

where V_+ is the positive sequence voltage before the fault. The negative and zero sequence voltages are zero because the system is balanced prior to the fault. The phase currents in the fault are then obtained by transformation:

$$I_{fpj} = \begin{bmatrix} I_{afj} \\ I_{bfj} \\ I_{cfj} \end{bmatrix} = TI_{fsj} \quad (6)$$

2.2 Voltages at the Busbars

The symmetrical component voltage at the faulted busbar j is given by:

$$V_{fsj} = \begin{bmatrix} V_{j+} \\ V_{j-} \\ V_{j0} \end{bmatrix} = \left(U + Z_{sjj}Y_{fs}\right)^{-1}V_{sj}^{\ 0} \quad (7)$$

The symmetrical component voltage at a busbar i for a fault at busbar j is given by:

$$V_{fsi} = \begin{bmatrix} V_{i+} \\ V_{i-} \\ V_{i0} \end{bmatrix} = V_{si}^{\ 0} - Z_{sij}Y_{fs}\left(U + Z_{sjj}Y_{fs}\right)^{-1}V_{sj}^{\ 0} \quad (8)$$

where $V_{si}^{\ 0} = \begin{bmatrix} V_{i+}^{\ 0} \\ 0 \\ 0 \end{bmatrix}$

gives the symmetrical component prefault voltages at busbar i. The negative and zero sequence prefault voltages are zero.

In equation (8), Z_{sij} is the ij^{th} components of the symmetrical component bus impedance matrix, the mutual terms for row i and column j (corresponding to busbars i and j)

$$Z_{sij} = \begin{bmatrix} Z_{sij+} & 0 & 0 \\ 0 & Z_{sij-} & 0 \\ 0 & 0 & Z_{sij0} \end{bmatrix}$$

The phase voltages in the fault, at busbar j, and at busbar i are then obtained by transformation

$$V_{fpj} = \begin{bmatrix} V_{afi} \\ V_{bfi} \\ V_{cfi} \end{bmatrix} = TV_{fsj} \quad \text{and} \quad V_{fpi} = \begin{bmatrix} V_{afpi} \\ V_{bfpi} \\ V_{cfpi} \end{bmatrix} = TV_{fpi} \quad (9)$$

2.3 Currents in Lines and Generators

The symmetrical component currents in a line between busbars i and j is given by:

$$I_{fsij} = Y_{fsij}\left(V_{fsi} - V_{fsj}\right) \quad (10)$$

where

$$Y_{fsij} = \begin{bmatrix} Y_{fsij+} & 0 & 0 \\ 0 & Y_{fsij-} & 0 \\ 0 & 0 & Y_{fsij0} \end{bmatrix}$$

is the symmetrical component admittance of the branch between busbars i and j.

The same equation applies to a generator where the source voltage is the prefault induced voltage and the receiving end busbar voltage is the postfault voltage at the busbar.

The phase currents in the branch are found by transformation:

$$I_{fpij} = \begin{bmatrix} I_{afij} \\ I_{bfij} \\ I_{cfij} \end{bmatrix} = TI_{fsij} \tag{11}$$

III. LINE-TO-LINE-TO-GROUND FAULT SIMULATION

Equation (4b) gives the symmetrical component fault admittance matrix for a line-to-line-to-ground fault, with equal fault admittances in the phases and ground. It is restated here for easy of reference:

$$Y_{fs} = \frac{Y}{9}\begin{bmatrix} 5 & -4 & -1 \\ -4 & 5 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

The value Y is the fault admittance in the faulted phases and ground.

The symmetrical component fault admittance matrix may be substituted in equation (5) to obtain the simplified expression of I_{fsj} given in equation (12), in which V_j^0 is the prefault voltage on bus bar j. The simplified formulation in equation (12) is useful for checking the accuracy of the symmetrical component currents in the fault when the general form is used.

$$I_{fsj} = \frac{\left(Z_{sjj-} + Z_{sjj0}\right)V_j^0}{Z_{sjj+}Z_{sjj0} + Z_{sjj+}Z_{sjj-} + Z_{sjj-}Z_{sjj0}} \begin{bmatrix} 1 \\ -\dfrac{Z_{sjj0}}{Z_{sjj-} + Z_{sjj0}} \\ -\dfrac{Z_{sjj-}}{Z_{sjj-} + Z_{sjj0}} \end{bmatrix} \tag{12}$$

The impedances required to simulate the line-to-line-to-ground fault are the impedances in the faulted phases and ground path. The impedances in the faulted phases and ground path are assumed equal to $5\times10^{-10}\ \Omega$. The open circuited phase a is simulated by a very high resistance of the order of $10^{50}\ \Omega$.

IV. COMPUTATION OF THE LINE-TO-LINE-TO-GROUND FAULTS ON REFERENCE AND ODD PHASES

A computer program that incorporates equations (1) to (12) has been developed to solve unbalanced faults for a general power system using the fault admittance matrix method. The program is applied on a simple power system comprising of three bus bars to solve for line-to-line-to-ground faults on the reference phase *a* and odd phases *b* and *c*. A simple system is chosen because it is easy to check the results against those that are obtained by hand.

4.1 Sample System

Figure 2 shows a simple three bus bar power system with one generator, one transformer and one transmission line. The system is configured based on the simple power system that Saadat uses [3].

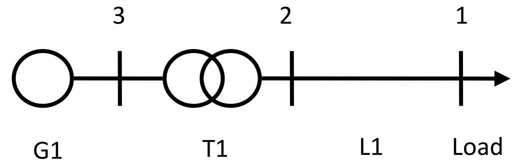

Figure 2: Sample Three Bus Bar System

The power system per unit data is given in Table 2, where the subscripts *1*, *2*, and *0* refer to the positive, negative and zero sequence values respectively. The neutral point of the generator is grounded through a zero impedance.

The transformer windings are delta connected on the low voltage side and earthed-star connected on the high voltage side, with the neutral solidly grounded. The phase shift of the transformer is 30°, i.e. from the generator side to the line side. Figure 3 shows the relationship of transformer voltages for a delta-star transformer connection Yd11 that has a 30° phase shift.

Table 2: Power System Data

Item	**S_{base}** (MVA)	**V_{base}** (kV)	**X_1** (pu)	**X_2** (pu)	**X_0** (pu)
G_1	100	20	0.15	0.15	0.05
T_1	100	20/220	0.1	0.1	0.1
L_1	100	220	0.25	0.25	0.7125

The computer program incorporates an input program that calculates the sequence admittance and impedance matrices and then assembles the symmetrical component bus impedance matrix for the power system. The symmetrical component bus impedance incorporates all the sequences values and has 3n rows and 3n columns where n is the number of bus bars. In general, the mutual terms between sequence values are zero as a three-phase power system is, by design, balanced.

The power system is assumed to be at no load before the occurrence of a fault. In practice the pre-fault conditions, established by a load flow study may be used. In developing the computer program the assumption of no load, and therefore voltages of

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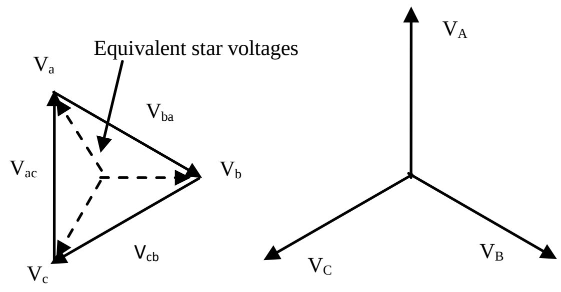

Figure 3: Delta-star Transformer Voltages for Yd11

1.0 per unit at the bus bars and in the generator, is adequate.

The line-to-line-to-ground faults are assumed to be at busbar *1*, the load busbar. They are described by the impedances in the respective phases and ground path.

The presence of the delta-earthed-star transformer poses a challenge in terms of its modelling. In the computer program the transformer is modelled in one of two ways; as a normal star-star connection, for the positive and negative sequence networks or as a delta star transformer with a phase shift. In the former model, the phase shifts are incorporated when assembling the sequence currents to obtain the phase values.

In particular on the delta connected side of the transformer the positive sequence currents' angles are increased by the phase shift while the angle of the negative sequence currents are reduced by the same value. The zero sequence currents, if any, are not affected by the phase shifts.

Both models for the delta star transformer give same results. The √3 line current factor is used to find the line currents on the delta side of the delta star transformer.

V. RESULTS AND DISCUSSIONS

5.1 Fault Simulation Impedances

The Thevenin's self sequence impedances of the network seen from the faulted bus bar are:

$$\begin{bmatrix} j0.5 & 0 & 0 \\ 0 & j0.5 & 0 \\ 0 & 0 & j0.8125 \end{bmatrix}$$

In the classical solution, the positive, negative and zero sequence currents due to a line-to-line-to-ground fault on phases *b* and *c* and ground summate to zero and are found by solving the parallel combination of the three sequence networks. Thus the sequence (positive, negative and zero) currents due to fault in the reference phase at the faulted bus bar are:

$$-j\begin{bmatrix} 1.2353 \\ -0.7647 \\ -0.4706 \end{bmatrix}$$

These sequence currents are compared with computed ones for the different line-to-line-to-ground faults, with the references on phase *a* and the odd phases *b* and *c*.

5.2 Simulation Results

The results obtained from the computer program are listed in Table 3. A summary of the transformer phase currents is given in Figures 4a, 4b and 4c for line-to-line-to-ground faults with references on phases *a*, *b* and *c* respectively.

5.3 Fault Admittance Matrix and Sequence Impedances at the Faulted Busbar.

The symmetrical component fault admittance matrix obtained from the program for the line-to-line-to-ground faults are in agreement with the theoretical values, obtained using equation (4b). The self sequence impedances at the faulted bus bar obtained from the program are equal to the theoretical values.

5.4 Fault Currents

The symmetrical component fault currents obtained from the program using equations (5) and (12) are in agreement. In particular, the positive, negative and zero sequence currents for the line-to-line-to-ground fault with reference to phase *a* summate to zero. The sum of the negative and zero sequence currents is equal and opposite that of the positive sequence current. This is consistent with the classical approach that connects the positive, negative and zero sequence networks in parallel.

When the line-to-line-to-ground fault is between phases c and a, with an odd phase reference of *b* the positive sequence current has a phase angle of -30° while the angles for the negative and zero currents are 270° and -30° respectively. The negative sequence component for the fault with reference *b* leads the respective component with the fault with reference phase *a* by 120° while the zero sequence current lags by 120°.

The results are consistent with theory, since the current symmetrical component constraints requirements for a line-to-line-to-ground fault with phase *b* reference are met, that is $\alpha^2 I_1 + \alpha I_2 + I_0 = 0$. Similarly when the line-to-line-to-ground fault is between phases *a* and *b* with an odd phase reference *c* the negative sequence component current leads the positive sequence current by 60° while the zero sequence current lags the positive sequence current by 60°. The symmetrical component sequence currents constraint is met; i.e. $\alpha I_1 + \alpha^2 I_2 + I_0 = 0$.

The phase currents in the fault obtained from the program are in agreement with the theoretical values. In particular, the currents in the healthy phases are zero and the currents in the ground path are of equal magnitude but displaced by 120° for the three cases.
The sum of the phase currents in the transmission line are equal to the current in the faults. Note that the current at the receiving end of the line is by convention considered as flowing into the line, rather than out of it.

Figures 4a, 4b and 4c summarise the transformer phase currents for the line-to-line-to-ground faults with references on phases *a*, *b* and *c* respectively. The currents in the transformer, on the line side, are equal to the currents in the line, after allowing for the sign changes due to convention. Note that the fault currents only flow in the windings of the faulted phases on the earthed-star connected side, and on the delta connected side as well. All the results satisfy the ampere-turn balance requirements of the transformer.

The currents at the sending end of the transformer, the delta connected side, flow into the leading faulted phase, i.e. phase *b* of the *bc* fault, phase *c* of the *ca* fault and phase *a* of the *ab* fault, and return in the remaining phases. In all the cases, the leading faulted phase carries twice the current in the other phases.

The phase fault currents flowing from the generator are equal to the phase currents into the transformer for all the three line-to-line-to-ground faults. For each fault the current in the leading faulted phase is twice the current in the other phases, which are equal in magnitude. All the generator phases have fault currents. It is a feature of the delta earthed-star connection that a line-to-line load on the star connected side is supplied from three phases on the delta side, both the transformer and generator.

5.5 Fault Voltages

The symmetrical component voltages at the fault point obtained from the program using equation (7) are in agreement with the theoretical values. In particular, the sequence voltages constraints in Table 1 are satisfied; The sequence positive, negative and zero component voltages for the phase *a* reference fault are equal, consistent with the concept of the three sequence networks being connected in parallel. When the line-to-line-to-ground fault is on the odd phases the respective symmetrical component voltage constraints are also satisfied; i.e. $V_{b1} = V_{b2} = V_{b0}$ that is $\alpha^2 V_1 = \alpha V_2 = V_0$ for the line-to-line-to ground *(cag)* fault with reference on odd

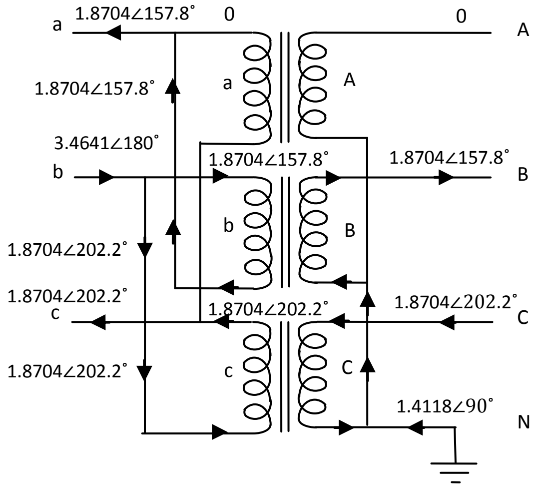

Figure 4a: Transformer Currents for a Line B to Line C to Ground Fault

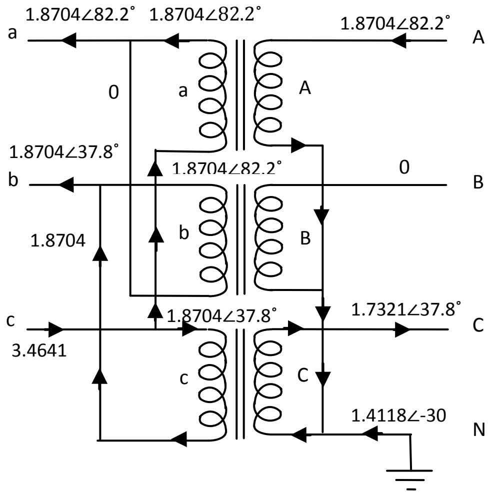

Figure 4b: Transformer Currents for a Line C to Line A to Ground Fault

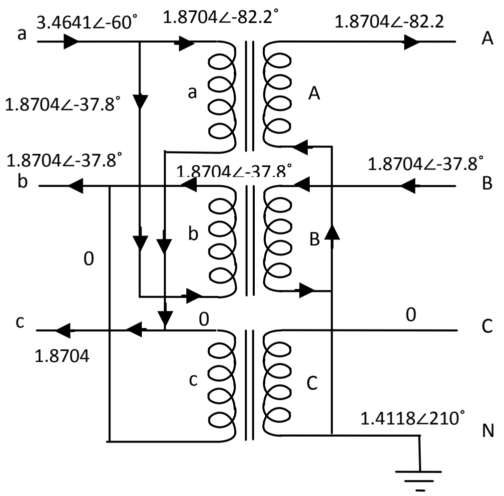

Figure 4c: Transformer Currents for a Line A to Line B to Ground Fault

phase *b*; and $V_{c1} = V_{c2} = V_{c0}$ that is $\alpha V_1 = \alpha^2 V_2 = V_0$ for the line-to-line-to-ground *(abg)* fault with reference on odd phase *c*.

Also note that at the faulted bus bar the magnitudes of the phase voltages of the healthy phases are at *1.1471* greater than unity, their prefault values, while the voltages in the faulted phases are zero.

The phase voltages at bus bar 2 show that the voltages in the faulted phases are *59%* of the prefault values while the voltages in the healthy phases are *93%* of the prefault values. At bus bar 3, the voltages in general lead the voltages at bus bar 2. In particular the voltages in the leading faulted phases lead the corresponding phase voltages at bus bar 2 by *42.3˚*. The increase in the phase shift between phase voltages of the faulted phases is due to the voltage drops in the transformer and generator. The voltages of the faulted phases that carry the return fault currents on the delta connected side of the transformer, i.e. phase *c* for the *bcg* fault and *a* for the *ca* fault and *b* for the *abg* fault, lead the voltages at bus bar *2* by *24.2˚*. The voltages in the healthy phases at bus bar 3 lead those of bus bar 2 by *23.5˚*.

VI. CONCLUSION

The general fault admittance method may be used to study line-to-line-to-ground faults on the reference phase *a* as well as with references on the odd phases *b* and *c*. The ability to handle line-to-line-to-ground faults with references on the odd phases makes it easier to study these faults as the method does not require the knowledge needed to translate the fault from the reference *a* phase to the odd phase references of phases *b* and c.

The line-to-line-to-ground fault is interesting for studying the delta earthed star transformer arrangement. It is seen that although only two phase carry the fault current on the earthed star side the currents on the delta connected side are in all three phases, although the winding on the delta side of the healthy phase does not carry any current. The current in the leading faulted phase on the delta side of the transformer is twice that in the other phases, with each phase carrying half of the return current.

Phase shifts in a delta earthed star connected transformer can be deduced from the results. The results give an insight in the effect that a delta earthed star transformer has on a power system during line-to-line faults.

The main advantage of the general fault admittance method is that the user is not required to know before-hand how the sequence networks should be connected at the fault point in order to obtain the sequence fault currents and voltages. The user can deduce the various relationships from the results. The method is therefore easier to use and teach than the classical approach in which each network is solved in isolation and then the results combined to obtain initially the sequence currents and voltages and then the phase quantities.

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Table 3: Simulation Results - Unbalanced Fault

General Fault Admittance Method – Delta-star Transformer Model
Number of busbars = 3
Number of transmission lines = 1
Number of transformers = 1
Number of generators = 1
Faulted busbar = 1
Fault type = 4

General Line to line to line to ground fault - Fault impedances

Phases BCg fault

Phase a	(R + j X)	1.0000e+050 +j 0.0000e+000
Phase b	(R + j X)	5.0000e-010 +j 0.0000e+000
Phase c	(R + j X)	5.0000e-010 +j 0.0000e+000
Ground	(R + j X)	5.0000e-010 +j 0.0000e+000

Phases CAg fault

Phase a	(R + j X)	5.0000e-010 +j 0.0000e+000
Phase b	(R + j X)	1.0000e+050 +j 0.0000e+000
Phase c	(R + j X)	5.0000e-010 +j 0.0000e+000
Ground	(R + j X)	5.0000e-010 +j 0.0000e+000

Phases ABg fault

Phase a	(R + j X)	5.0000e-010 +j 0.0000e+000
Phase b	(R + j X)	5.0000e-010 +j 0.0000e+000
Phase c	(R + j X)	1.0000e+050 +j 0.0000e+000
Ground	(R + j X)	5.0000e-010 +j 0.0000e+000

Fault Admittance Matrix - Real and Imaginary Parts of Fault Admittance Matrix

Phases BCg fault

1.1111e+009 +j 0.0000e+000	-8.8889e+008 +j 0.0000e+	-2.2222e+008 +j 0.0000e+000
-8.8889e+008 +j 0.0000e+000	1.1111e+009 +j 0.0000e+000	-2.2222e+008 +j 0.0000e+000
-2.2222e+008 +j 0.0000e+000	-2.2222e+008 +j 0.0000e+000	4.4444e+008 +j 0.0000e+000

Phases CAg fault

1.1111e+009 +j 0.0000e+000	4.4444e+008 +j 7.6980e+008	1.1111e+008 +j -1.9245e+008
4.4444e+008 +j -7.6980e+008	1.1111e+009 +j 0.0000e+000	1.1111e+008 +j 1.9245e+008
1.1111e+008 +j 1.9245e+008	1.1111e+008 +j -1.9245e+008	4.4444e+008 +j 0.0000e+000

Phases ABg fault

1.1111e+009 +j 0.0000e+000	4.4444e+008 +j -7.6980e+008	1.1111e+008 +j 1.9245e+008
4.4444e+008 +j 7.6980e+008	1.1111e+009 +j 0.0000e+000	1.1111e+008 +j -1.9245e+008
1.1111e+008 +j -1.9245e+008	1.1111e+008 +j 1.9245e+008	4.4444e+008 +j 0.0000e+000

Thevenin's Symmetrical Component Impedance Matrix of Faulted Busbar - Real and imaginary parts

Phases BCg fault

0.0000 +j 0.5000	0.0000 +j 0.0000	0.0000 +j 0.0000
0.0000 +j 0.0000	0.0000 +j 0.5000	0.0000 +j 0.0000
0.0000 +j 0.0000	0.0000 +j 0.0000	0.0000 +j 0.8125

Phases ABg fault

0.0000 +j 0.5000	0.0000 +j 0.0000	0.0000 +j 0.0000
0.0000 +j 0.0000	0.0000 +j 0.5000	0.0000 +j 0.0000
0.0000 +j 0.0000	0.0000 +j 0.0000	0.0000 +j 0.8125

Phases CAg fault

0.0000 +j 0.5000	0.0000 +j 0.0000	0.0000 +j 0.0000
0.0000 +j 0.0000	0.0000 +j 0.5000	0.0000 +j 0.0000
0.0000 +j 0.0000	0.0000 +j 0.0000	0.0000 +j 0.8125

Fault Current in Symmetrical Components - Real, Imaginary, Magnitude and Angle

Phase BCg fault

	Real	Imag	Magn	Angle [Deg]
+ve	0.0000	-1.2353	1.2353	-90.0000
-ve	0.0000	0.7647	0.7647	90.0000
zero	0.0000	0.4706	0.4706	90.0000

Phases CAg fault

+ve	0.0000	-1.2353	1.2353	-90.0000
-ve	-0.6623	-0.3824	0.7647	210.0000
zero	0.4075	-0.2353	0.4706	-30.0000

Phases ABg fault

+ve	-0.0000	-1.2353	1.2353	270.0000
-ve	0.6623	-0.3824	0.7647	-30.0000
zero	-0.4075	-0.2353	0.4706	210.0000

Fault current in phase components - Rectangular and Polar Coordinates

Phases BCg fault

	Real	Imag	Magn	Angle [Deg]
Phase a	0.0000	+j 0.0000	0.0000	90.0000
Phase b	-1.7321	+j 0.7059	1.8704	157.8271
Phase c	1.7321	+j 0.7059	1.8704	22.1729
Ground	0.0000	+j 1.4118	1.4118	90.0000

Phases CAg fault

Phase a	-0.2547	+j -1.8529	1.8704	262.1729
Phase b	0.0000	+j -0.0000	0.0000	-65.7617
Phase c	1.4773	+j 1.1471	1.8704	37.8271
Ground	1.2226	+j -0.7059	1.4118	-30.0000

Phases ABg fault

Phase a	0.2547	+j -1.8529	1.8704	-82.1729
Phase b	-1.4773	+j 1.1471	1.8704	142.1729
Phase c	0.0000	+j -0.0000	0.0000	-88.9821
Ground	-1.2226	+j -0.7059	1.4118	210.0000

Symmetrical Component Voltages at Faulted Busbar - Rectangular and Polar Coordinates

Phases BCg fault

	Real	Imag	Magn	Angle [Deg]
+ve	0.3824	0.0000	0.3824	0.0000
-ve	0.3824	-0.0000	0.3824	-0.0000
zero	0.3824	-0.0000	0.3824	-0.0000

Phases CAg fault

+ve	0.3824	0.0000	0.3824	0.0000
-ve	-0.1912	0.3311	0.3824	120.0000
zero	-0.1912	-0.3311	0.3824	240.0000

Phases CAg fault

+ve	0.3824	0.0000	0.3824	0.0000
-ve	-0.1912	-0.3311	0.3824	240.0000
zero	-0.1912	0.3311	0.3824	120.0000

Phase Voltages at Faulted Busbar

Phases BCg fault

	Real	Imag	Magn	Angle [Deg]
Phase a	1.1471	0.0000	1.1471	0.0000
Phase b	-0.0000	-0.0000	0.0000	218.9062
Phase c	-0.0000	0.0000	0.0000	143.3063

Phases CAg fault

Phase a	0.0000	-0.0000	0.0000	-36.1654
Phase b	-0.573	-0.9934	1.1471	240.0000
Phase c	-0.0000	0.0000	0.0000	154.9240

Phases ABg fault

Phase a	-0.0000	0.0000	0.0000	176.2053
Phase b	0.0000	-0.0000	0.0000	-22.9986
Phase c	-0.5735	0.9934	1.1471	120.0000

Postfault Voltages at Busbar number = 1

Phases BCg fault

	Real	Imag	Magn	Angle [Deg]
Phase a	1.1471	0.0000	1.1471	0.0000
Phase b	-0.0000	-0.0000	0.0000	218.9062
Phase c	-0.0000	0.0000	0.0000	143.3063

Phases CAg fault

Phase a	0.0000	-0.0000	0.0000	-36.1654
Phase b	-0.5735	-0.9934	1.1471	240.0000
Phase c	-0.0000	0.0000	0.0000	154.9240

Phases ABg fault

Phase a	-0.0000	0.0000	0.0000	176.2053
Phase b	0.0000	-0.0000	0.0000	-22.9986
Phase c	-0.5735	0.9934	1.1471	120.0000

Postfault Voltages at Busbar number = 2

Phases BCg fault

	Real	Imag	Magn	Angle [Deg]
Phase a	0.9294	0.0000	0.9294	0.0000
Phase b	-0.3941	-0.4330	0.5855	227.6923
Phase c	-0.3941	0.4330	0.5855	132.3077

Phases CAg fault

Phase a	0.5721	0.1248	0.5855	12.3077
Phase b	-0.4647	-0.8049	0.9294	240.0000
Phase c	-0.1779	0.5578	0.5855	107.6923

Phases ABg fault

Phase a	0.5721	-0.1248	0.5855	-12.3077
Phase b	-0.1779	-0.5578	0.5855	252.3077
Phase c	-0.4647	0.8049	0.9294	120.0000

Postfault Voltages at Busbar number = 3

Phases BCg fault

	Real	Imag	Magn	Angle [Deg]
Phase a	0.8049	0.3500	0.8777	23.5013
Phase b	0.0000	-0.7000	0.7000	-90.0000
Phase c	-0.8049	0.3500	0.8777	156.4987

Phases CAg fault

Phase a	0.7056	0.5221	0.8777	36.4987
Phase b	-0.0993	-0.8721	0.8777	263.5013
Phase c	-0.6062	0.3500	0.7000	150.0000

Phases ABg fault

Phase a	0.6062	0.3500	0.7000	30.0000
Phase b	0.0993	-0.8721	0.8777	-83.5013
Phase c	-0.7056	0.5221	0.8777	143.5013

Postfault Currents in Lines

Phases BCg fault

			Phase a		Phase b		Phase c	
Line No.	SE Bus	RE Bus	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.
1	2	1	0.0000	90.0000	1.8704	157.8271	1.8704	22.1729
1	1	2	0.0000	-90.0000	1.8704	-22.1729	1.8704	202.1729
Phases CAg fault								
1	2	1	1.8704	262.1729	0.0000	-65.7617	1.8704	37.8271
1	1	2	1.8704	82.1729	0.0000	114.2383	1.8704	217.8271
Phases ABg fault								
1	2	1	1.8704	-82.1729	1.8704	142.1729	0.0000	-88.9821
1	1	2	1.8704	97.8271	1.8704	-37.8271	0.0000	91.0179

Postfault Currents in Transformers

Phases BCg fault

			Phase a		Phase b		Phase c	
Transf No.	SE Bus	RE Bus	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.	Current Magn Deg.	Current Angle
1	3	2	1.8704	-22.1729	3.4641	180.0000	1.8704	22.1729
1	2	3	0.0000	118.9409	1.8704	-22.1729	1.8704	202.1729

Neutral Current at Receiving end

Real	Imag	Magn	Deg
0.0000	-1.4118	1.4118	-90.0000

Link Currents in Delta Connection at Sending End

			Phase a		Phase b		Phase c	
No.	Bus	Bus	Magn	Angle Deg.	Magn	Angle Deg.	Magn	Angle Deg.
1	3	2	1.8704	-22.1729	1.8704	202.1729	0.0000	229.6185

Phases CAg fault

			Phase a		Phase b		Phase c	
Transf No.	SE Bus	RE Bus	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.
1	3	2	1.8704	262.1729	1.8704	217.8271	3.4641	60.0000
1	2	3	1.8704	82.1729	0.0000	2.0245	1.8704	217.8271

Neutral Current at Receiving End

Real	Imag	Magn	Deg
-1.2226	0.7059	1.4118	150.0000

Link Currents in Delta Connection at Sending End

			Phase a		Phase b		Phase c	
No.	Bus	Bus	Magn	Angle Deg.	Magn	Angle Deg.	Magn	Angle Deg.
1	3	2	0.0000	109.7578	1.8704	217.8271	1.8704	82.1729

Phases ABg fault

			Phase a		Phase b		Phase c	
Transf No.	SE Bus	RE Bus	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.
1	3	2	3.4641	-60.0000	1.8704	142.1729	1.8704	97.8271
1	2	3	1.8704	97.8271	1.8704	-37.8271	0.0000	235.8617

Neutral Current at Receiving End

Real	Imag	Magn	Deg
1.2226	0.7059	1.4118	30.0000

Link Currents in Delta Connection at Sending End

			Phase a		Phase b		Phase c	
Gen No.	SE Bus	RE Bus	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.
1	3	2	1.8704	-37.8271	0.0000	-7.5498	1.8704	97.8271

Postfault Currents in Generators

Phases BCg fault

			Phase a		Phase b		Phase c	
Gen No.	SE Bus	RE Bus	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.
1	4	3	1.8704	-22.1729	3.4641	180.0000	1.8704	22.1729

Generator Neutral current

Real	Imag	Magn	Deg
-0.0000	0.0000	0.0000	109.9831

Phases CAg fault

			Phase a		Phase b		Phase c	
Gen No.	SE Bus	RE Bus	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.
1	4	3	1.8704	262.1729	1.8704	217.8271	3.4641	60.0000

Generator Neutral current

Real	Imag	Magn	Deg
0.0000	0.0000	0.0000	0.0000

Phases ABg fault

			Phase a		Phase b		Phase c	
Gen No.	SE Bus	RE Bus	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.	Current Magn	Current Angle Deg.
1	4	3	3.4641	-60.0000	1.8704	142.1729	1.8704	97.8271

Generator Neutral current

Real	Imag	Magn	Deg
0.0000	0.0000	0.0000	53.1301